Poromechanics – Biot Centennial (1905–2005) – Abousleiman, Cheng & Ulm (eds)
© 2005 Taylor & Francis Group, London, ISBN 04 1538 041 3

Elastoplastic constitutive model for unsaturated soil-steel interface

T.B. Hamid & G.A. Miller
University of Oklahoma, Norman, Oklahoma, USA

ABSTRACT: A constitutive model based on the elastoplasticity theory developed by Geiser et al. (2000) is modified to predict the behavior of interfaces between unsaturated soil and steel. The proposed model is formulated using two stress variables, net normal stress and matric suction. The predictions of the proposed model are verified with respect to test data on interfaces with different suction, net normal stress, and roughness. The model parameters were determined from constant suction direct shear box tests and were used in the prediction of the unsaturated interface behavior. The model is able to reproduce the important features of unsaturated interface behavior as observed in laboratory testing. For the condition where matric suction is zero, the proposed model becomes the Hierarchical Single Surface (HISS) model as proposed by Navayogarajah et al. (1992).

1 INTRODUCTION

Load transfer between a particulate material and a solid surface occurs across an interface (Dietz 2002). The interface involves a thin layer of material with a defined thickness adjacent to the contact surface. Interface behavior has been studied by several researchers (e.g., Boulon & Nova 1990, Desai & Ma 1992, Dietz 2002). Most of the interface models were developed based on laboratory results from sand testing and various construction materials, and do not include the effect of pore water and air pressure on the behavior of the interface. There are many structures that are founded on the partially saturated soil; the shear strength of the unsaturated interface is governed by two stress state variables, net normal stress (σ_n–u_a) and matric suction (u_a–u_w) (Hamid 2005). Net normal stress is the difference between the total normal stress and pore air pressure (σ_n–u_a) and matric suction is the difference between pore air and pore water pressure (u_a–u_w). In this paper an elastoplastic constitutive model for the interface between unsaturated soil and steel is presented utilizing two stress state variables, σ_n–u_a and u_a–u_w.

2 GENERAL DESCRIPTION

The following generalized yield function was proposed by Geiser et al. (2000) to describe the yielding of unsaturated soil at constant suction:

$$F=\frac{j_{2D}}{p_a^2}-\left[-\alpha(s)\left(\frac{j_1+R(s)}{p_a}\right)^n+\gamma\left(\frac{j_1+R(s)}{p_a}\right)^2\right]F_s \qquad (1)$$

where:
j_{2D} = second invariant of the deviatoric stress tensor,
p_a = atmospheric pressure,
$\alpha(s)$ = hardening function,
j_1' = first invariant of the saturated effective stress tensor,
$R(s)$ = bonding stress,
$F_s = 1-\beta\overline{S}_r$,
$\overline{S}_r=\sqrt{27/2}(j_{3D}\,j_{2D}^{-3/2})$,
j_{3D} = third stress invariant of the deviatoric stress tensor, and γ and β are ultimate state parameters.

In this paper a specialized form of Equation 1 is developed to describe the yielding of interfaces between unsaturated soil and steel. To modify Equation 1 analogies between unsaturated soil (Cui and Delage, 1996) and unsaturated interface (Hamid, 2005) test results were developed using the same procedure as described by Boulon & Nova (1990) and Desai & Fishman (1991) as shown in Table 1. In the proposed model, hardening and potential functions are described in terms of suction and net normal stress; however, for the case when u_a–u_w = 0 these functions take the same form as proposed by Navayogarajah et al. (1992). Therefore, the proposed model represents a

Table 1. Analogous quantities for unsaturated soil and interfaces.

Unsaturated soil	Unsaturated interface
q	τ
$p=(\sigma_1+\sigma_2+\sigma_3/3)-u_a$	$\sigma_{net}=\sigma_n-u_a$
ε_v	v
ε_s	u
u_a–u_w	u_a–u_w

general case of the model developed by Navayogarajah et al. (1992) for interface between dry sand and steel.

3 SPECIALIZATION OF UNSATURATED SOIL MODEL

3.1 *Yield function*

Following the analogies between unsaturated soil and unsaturated interface, a yield function for unsaturated soil-steel interface was obtained as a special case of Equation 1:

$$F=\tau^2+\alpha(s)[\sigma_{net}+R(s)]^n-\gamma(s)[\sigma_{net}+R(s)]^2 \tag{2}$$

Where: τ is the shear stress, $\sigma_{net}=\sigma_n-u_a$ and n is a phase change parameter. The (s) indicates dependency of parameter on suction (u_a-u_w). The $R(s)$ represents the increase in strength of unsaturated interface with increase in suction; it can be considered as the value of effective cohesion in the net normal stress-shear stress plane. The parameter $\gamma(s)$ in Equation 2 is given as:

$$\gamma(s)=\left[\frac{\tau}{(\sigma_{net}+R(s))}\right]^2 \tag{3}$$

For solids the parameter F_s in Equation 1 controls the shape of the yield functions plotted in principal stress space. However, for interfaces in unsaturated soil, the function is plotted in net normal stress (σ_{net}) versus shear stress (τ) space. Therefore, F_s is taken as unity. Parameter n is related to the state of stress at which transition from compaction to dilation occurs or at which the change in the volume is zero.

The $\alpha(s)$ is a hardening function that defines the evolution of the yield surface during deformation and is described in the next section.

3.2 *Hardening function*

Hardening function, $\alpha(s)$, is defined as follows by two conditions.
Condition I:
when $\xi_D<\xi_D^*$

$$\alpha(s)=\gamma(s)\exp(-a(s)\xi_v)\left(\frac{\xi_D^*-\xi_D}{\xi_D^*}\right)^{b(s)} \tag{4}$$

Condition II:
when $\xi_D\geq\xi_D^*$

$$\alpha(s)=0 \tag{5}$$

where: $\xi_v=\int|dv^p|$ and $\xi_D=\int|du^p|$ are the plastic volumetric and shear displacement trajectories, respectively. ξ_D^* is the value of ξ_D where shear stress is maximum; dv^p & du^p indicate the plastic part of the total displacement normal and tangential to the shearing surface, respectively.
Parameters a, b, n, and ξ_D^* are functions of $R(s)$ and roughness ratio (R_n), which is defined as follows:

$$R_n=R_{max}/D_{50} \tag{6}$$

R_{max} is the maximum distance between peak and valley of the rough surface and D_{50} is the diameter of the soil grain corresponding to fifty percent finer.

3.3 *Potential function*

The following potential function is proposed as the nonassociative flow rule.

$$Q=\tau^2+\alpha_Q(s)[\sigma+R(s)]^n-\gamma(s)[\sigma+R(s)]^2 \tag{7}$$

Where the function $\alpha_Q(s)$ is given by the following expression.

$$\alpha_Q(s)=\alpha(s)+\alpha_{ph}\left(1-\frac{\alpha(s)}{\alpha_i}\right)\left[1-\kappa\left(1-\frac{D}{D_u}\right)\right] \tag{8}$$

Where α_{ph} and α_i are the values of $\alpha(s)$ at the phase change point and initiation of nonassociativeness, respectively, and their values are given as follows (Navayogarajah et al. 1992):

$$\alpha_{ph}=\frac{2\gamma}{n}\sigma_{net}^{2-n} \tag{9}$$

$$\alpha_i=\gamma\sigma_{net}^{2-n} \tag{10}$$

In Equation 8, κ is a material parameter and is related to the roughness ratio, net normal stress, and suction. Note that $R(s)$ vanishes for the case when $u_a-u_w=0$ and for this condition Equations 2 to 8 take the same form as proposed by Navayogarajah et al. (1992). Most of the parameters in Equations 2 to 8 are dependent on suction and net normal stress as well as on roughness.

In Equation 8, $D_u=\frac{\tau_p-\tau_r}{\tau_p}$; τ_p and τ_r are the peak and residual shear stress, respectively. D is a damage function that defines the post peak behavior of interface and is defined in the next section.

3.4 *Post-peak behavior*

Strain softening was observed in all interface tests conducted during the current study on clayey silt. Strain softening behavior was more pronounced for higher suction values. To model the strain softening effect the disturbed state concept is employed. Desai & Ma (1992) and Navayogarajah et al. (1992) have already used the disturbed state concept to model the interface behavior without the influence of suction. In the disturbed state concept, observed or average stress is defined as the sum of the stress in the relative part and stress in the fully adjusted part. The following relationship for the observed stresses is proposed:

$$\begin{Bmatrix} \sigma_{net} \\ \tau \end{Bmatrix} = (1-D)\begin{Bmatrix} \sigma^{t}_{net} \\ \tau^{t} \end{Bmatrix} + D\begin{Bmatrix} \sigma^{t}_{net} \\ 0 \end{Bmatrix} \tag{11}$$

superscript t in equation (11) indicates the intact part of the interface.

Damage function D is given as follows:

For $\xi_D < \xi_D^*$

$D=0$ (12)

and

For $\xi_D \geq \xi_D^*$

$$D = D_u - D_u \exp\left[-\left(\xi_D - \xi_D^*\right)^2\right] \tag{13}$$

4 BACK PREDICTIONS

4.1 *Incremental stress-displacement relations*

The total incremental relative displacement, in normal and tangential direction, is decomposed into elastic (recoverable) part and plastic (nonrecoverable) part as follows:

$$dv = dv^e + dv^p \tag{14}$$

$$du = du^e + du^p \tag{15}$$

or

$$\{dU\} = \{dU^e\} + \{dU^p\} \tag{16}$$

Where $\{dU\}^T = (dv, du)$; dv and du are the total relative displacements normal and tangential to the shearing plane, respectively. Superscripts e and p indicate the elastic part and plastic part of the displacements, respectively.

The consistency condition (i.e., $dF = 0$) yields the following expression

$$dF = \frac{\partial F}{\partial \sigma_{net}} d\sigma_{net} + \frac{\partial F}{\partial \xi_D} d\xi_D + \frac{dF}{d\xi_v} d\xi_v = 0 \tag{17}$$

Elastic displacements are related to the stresses by the following relation:

$$\begin{Bmatrix} d\sigma_{net} \\ d\tau \end{Bmatrix} = \begin{bmatrix} K_n & 0 \\ 0 & K_s \end{bmatrix} \begin{Bmatrix} dv^e \\ du^e \end{Bmatrix} \tag{18}$$

or

$$d\sigma = C^e dU^e$$

Where C^e = elastic constitutive matrix of interface, K_n and K_s = elastic normal and shear stiffness of the interface, respectively. It is assumed that elastic normal and shear behavior of the interface are uncoupled.

The permanent relative displacement due to sliding and volume change displacement are related to the plastic potential function by the flow rule:

$$\begin{Bmatrix} dv^p \\ du^p \end{Bmatrix} = \lambda \begin{bmatrix} \partial Q/\partial \sigma \\ \partial Q/\partial \tau \end{bmatrix} \tag{19}$$

where Q is the potential function and its value is given in Equation 7. The λ is a plastic multiplier and its value is given as follows:

$$\left.\begin{matrix} \lambda = 0 \text{ if } F<0 \text{ or } dF<0 \\ \lambda > 0 \text{ if } F=0 \text{ or } dF=0 \end{matrix}\right\} \tag{20}$$

Combining Equations 14–19 and eliminating the λ the incremental stress-relative displacement relation can be written as:

$$\{d\sigma\} = \left[C^e\right]\left(1 - \frac{\left(\frac{\partial F}{\partial\{d\sigma\}}\right)^T \left[C^e\right]\left(\frac{\partial Q}{\partial\{d\sigma\}}\right)}{\left(\frac{\partial F}{\partial\{d\sigma\}}\right)^T \left[C^e\right]\left(\frac{\partial Q}{\partial\{d\sigma\}}\right) - H}\right)\{dU\} \tag{21}$$

In Equation 21 the parameter H is the plastic modulus and is given by

$$H = \left(\frac{\partial F}{\partial \xi_v}\right)\left(\left|\frac{\partial Q}{\partial \sigma}\right|\right) + \left(\frac{\partial F}{\partial \xi_D}\right)\left(\left|\frac{\partial Q}{\partial \tau}\right|\right) \tag{22}$$

H will be zero if no hardening is present in the model. In other words when the ratio of net normal stress to shear stress becomes constant the model does not include any hardening.

4.2 *Laboratory testing*

Performance of the proposed model was checked by comparing the predicted results with the laboratory test results obtained by using the unsaturated interface direct shear device. The Unsaturated Interface

Direct Shear Device (UIDSD) was designed and constructed at The University of Oklahoma and details are given by Hamid (2005). Locally available soil, Minco silt (clayey silt), was used to examine the behavior of unsaturated soil-steel interface behavior. All the samples were tested at an initial dry density of 15.7 kN/m^3 and initial water content 20 ± 1%. Stainless steel plates of two roughnesses (smooth and rough) were used as counterfaces. Samples were tested under 20, 50 and 100 kPa suction values under constant net normal stress condition. Three net normal stresses of 105, 140, and 210 kPa were used for each suction value.

Based on experimental results the following observations can be made about the behavior of the unsaturated soil-steel interfaces:

1) Peak shear strength of unsaturated soil-steel interface (smooth and rough both) increased with increasing suction.
2) Residual shear strength of unsaturated interface (smooth and rough) did not change with the suction.
3) Peak and residual shear stress of unsaturated interface increased with increasing net normal stress.
4) Strain softening effect was more pronounced for higher suction and higher net normal stress.
5) Suction effect on peak shear strength was more pronounced in the rough interface than smooth.
6) Rough interface initially compresses and then dilates or achieves steady state.
7) Smooth interface initially showed compression and then achieved steady state.

4.3 *Verification of model*

Model parameters used for back predictions are given in Table 2. Model parameters $R(s)$, γ, ξ_D^*, κ, a, and b are functions of roughness ratio (R_n), u_a–u_w, and σ_n–u_a. Test results corresponding to u_a–u_w = 50 kPa and σ_n–u_a = 140 kPa (Figure 1) were not used in the determination of model parameters. Details for the determination of model parameters and their functional forms in terms of net normal stress, matric suction and roughness are given by Hamid (2005).

The comparison between the experimental results and predictions shows that the proposed model is able to represent the main behavior of unsaturated interfaces as described above.

Comparison of Figures 1a and 2a shows that as suction (u_a–u_w) increased from 50 kPa to 100 kPa, the shear stress of rough interface increased from 117 kPa to 130 kPa. Back predicted results shown on the same figures illustrate the capability of the model to capture the effect of suction on the rough interface.

Figures 1b and 2b show the volume change behavior of rough interface and the proposed model back predicted the experiment results very well.

Table 2. Values of model parameters for different values of u_a–u_w, and σ_n–u_a for rough and smooth.

	Rough				Smooth	
σ_n–u_a (kPa)	140	105	140	210	105	105
u_a–u_w (kPa)	50	100	100	100	20	100
Rn	7.6	7.6	7.6	7.6	0.05	0.05
R(s)	9.70	29.65	29.65	29.65	37.20	69.13
γ	0.581	0.581	0.581	0.581	0.080	0.080
ξ_D^* (mm)	0.764	0.783	0.783	0.783	0.072	0.102
κ	0.089	0.589	0.730	1.00	−0.23	−0.23
n	4.0	4.0	4.0	4.0	7.0	7.0
a	17.4	17.4	17.4	17.4	56.0	56.0
b	1.0	1.0	1.0	1.0	1.0	1.0

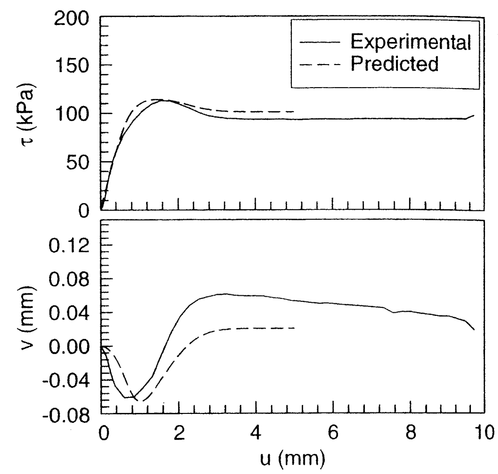

Figure 1. Comparison of observations and predictions for σ_n–u_a = 140 kPa and u_a–u_w = 50 kPa for rough interface.

Effect of net normal stress (σ_n–u_a) on the shear strength and volume change behavior of a rough interface is shown in Figure 3 along with the predicted results. The proposed model was able to back predict the experimental results satisfactorily.

Comparison of Figures 4a and 5a illustrates the effect of u_a–u_w on the shear strength of the smooth interface. The smooth interface showed stick-slip behavior after reaching maximum shear strength. No attempt was made to capture the stick-slip behavior in the proposed model. However, the model is able to back predict the maximum shear strength and approximately the lowest shear strength value after the maximum shear stress.

As opposed to the rough interface, the smooth interface achieved steady state after the initial compression and it did not show any dilation. Comparison

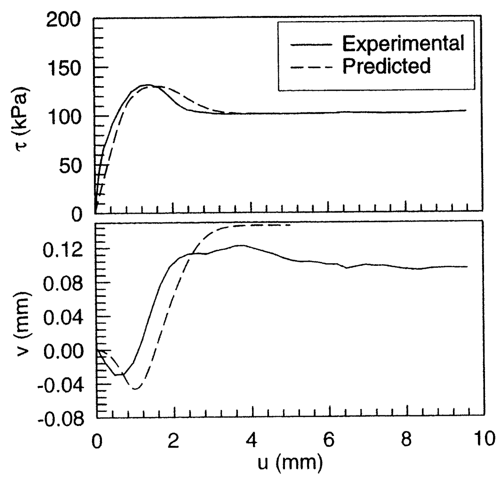

Figure 2. Comparison of observations and predictions for σ_n–u_a = 140 kPa and u_a–u_w = 100 kPa for rough interface.

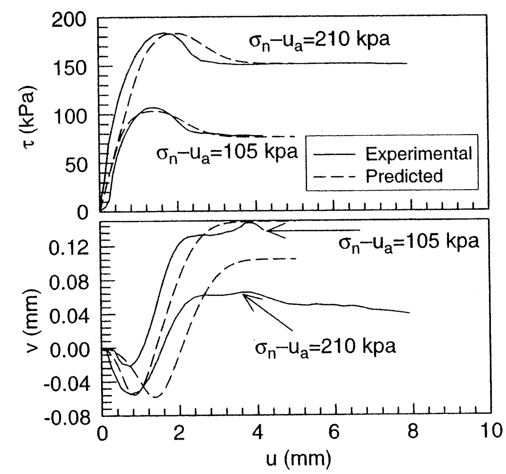

Figure 3. Comparison of observations and predictions for u_a–u_w = 100 kPa for rough interface.

of experimental results and predictions show that the proposed model captured the volume change behavior of smooth interface very well.

5 CONCLUSIONS

Performance of a proposed constitutive model is examined by using unsaturated soil-steel interface direct shear test results. The model is able to predict the important characteristics of shear strength and volume change behavior of an unsaturated interface. Predicted results compare well with the experimental results.

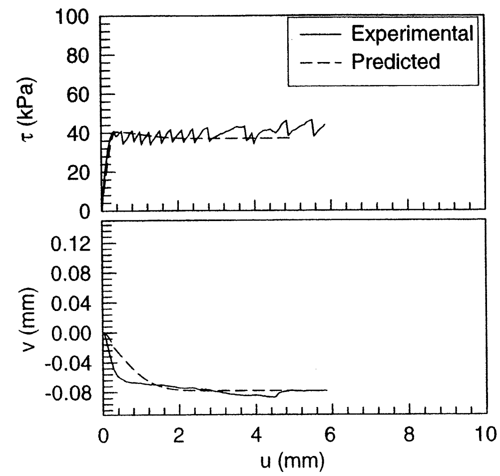

Figure 4. Comparison of observations and predictions for σ_n–u_a = 105 kPa and u_a–u_w = 20 kPa for smooth interface.

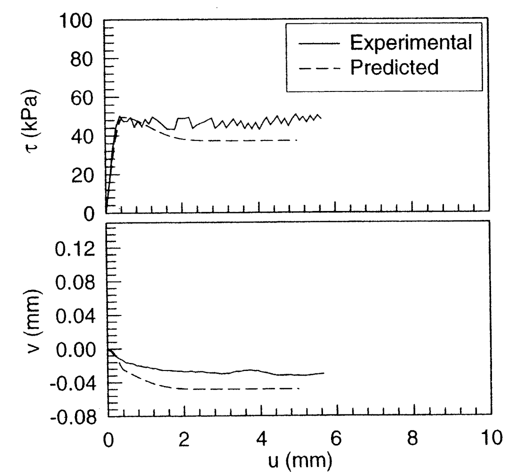

Figure 5. Comparison of observations and predictions for σ_n–u_a = 105 kPa and u_a–u_w = 100 kPa for smooth interface.

Model parameters were determined from constant net normal stress and constant suction unsaturated interface direct shear tests. The disturbed state concept is used to represent the strain softening behavior of an unsaturated interface.

REFERENCES

Boulon, M. & Nova, R. 1990. Modelling of soil-structure interface behaviour a comparison between elastoplastic and rate type laws. Computer and Geotechnics. 9: 21–46.

Cui, Y.J. & Delage, P. 1996.Yielding and plastic behaviour of an unsaturated compacted silt. Geotechnique. 46(2): 291–311.

Desai, C. S. & Fishman, K. L. 1991. Plasticity – based constitutive model with associated testing for joints. International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts. 28(1): 15–26.

Desai, C. S. & Ma, Y. 1992. Modelling of joints and interfaces using the disturbed-state concept, International Journal of Numerical and Analytical Methods in Geomechanics, 16: 623–653.

Dietz, M. S. 2000. Experimental observations of interface friction. Constitutive and Centrifuge Modelling: Two Extremes, Springman (ed.) 2002 Swets & Zeitlinger, Lisse.

Geiser, F. Laloui, L. & Vulliet, L. 2000. Modelling the behaviour of unsaturated silt. Experimental Evidence and Theoretical Approaches in Unsaturated Soil. 155–175. Rotterdam:Balkema.

Hamid, T.B. 2005. Testing and modeling of unsaturated interfaces. Doctoral Dissertation. School of Civil Engineering and Environmental Science. University of Oklahoma.

Navayogarajah, N. Desai, C. S. & Kiousis, P. D. 1992. Hierarchical single surface model for static and cyclic behavior of interfaces. Journal of Engineering Mechanics. 118(5): 990–1011.